 UG — BCA (F – 2001)

2022(New)

41/70

Time : 3 hours

Full Marks : 70

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Groups as directed.

Group – A

1. Answer any **four** questions of the following :

(a) Define subgroup of a group. Prove that a non-empty set H of a group G is a subgroup if and only if $\forall$ a, b $\in$ H $\Rightarrow$ $ab^{-1} \in H$. 10

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(b) Prove that every field is an integral domain. 10

(c) For a finite dimensional vector space V(F), any two bases have same number of elements. 10

(d) Show that the vectors (2, 1, 4), (1, −1, 2) and (3, 1, −2) form a basis for V_3. 10

(e) Show that the following system of linear equations is not consistent : 10

$$x - 4y + 7z = 14$$

$$3x + 8y - 2z = 13$$

$$7x - 8y + 26z = 5$$

(f) Verify the Caley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. 10

(g) Prove that the points (0, −1, 0), (2, 1, −1), (1, 1, 1) and (1, 2, 4) are coplanar. 10

(h) Find the equation of the plane through the point (−1, 3, 2) and perpendicular to the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 9$. 10

Group – B

Answer all questions of the following :

2. In the multiplicative group $G = \{1, \omega, \omega^2\}$, find the order of ω and ω^2, 3

where $\omega = \dfrac{-1 + \sqrt{3}i}{2}$.

3. In the multiplicative group $G = \{1, -1, i, -i\}$, find the order of each element of G.

4. Define subgroup of a group. 3

5. Prove that in a ring R, $(-a)(-b) = ab; \forall a, b \in R$. 3

6. Define an integral domain. 3

7. Write down a basis for R^3. 3

8. State the elementary row operations in a matrix. 3

9. Define a matrix polynomial. 3

10. Prove that the points (0, 4, 1), (2, 3, −1), (4, 5, 0) and (2, 6, 2) are the vertices of a square. 3

11. Define direction cosine of a line. 3

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2021

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*Answer any **four** questions in which Q. No. 1 is compulsory.*

1. Answer **all** questions : 2×10 = 20

(a) In the multiplicative group $G = (1, \omega, \omega^2)$, find the inverse of ω^2.

(b) In the multiplicative group $G = (1, -1, i, -i)$, find the order of i.

(c) State Lagrange's theorem.

(d) Prove that in a ring R, $a(-b) = -ab$; $\forall\, a\, b \in R$.

(e) Define a field.

(f) Write down a basis for R^4.

(g) Write down a matrix of order three in Echelon form.

(h) Define a matrix polynomial.

(i) Determine the relation between Cartesian and Polar spherical coordinates of a point.

(j) Find the direction cosines of a line passing through the points (4, 5, 0) and (2, 6, 2).

2. (a) Prove that in a group G, $(ab)^{-1} - b^{-1} a^{-1}$; $\forall\, a, b \in G$ 10

(b) Prove that the identity element in a group is unique. 10

3. (a) Prove that a finite integral domain is a field. 10

(b) Prove that the power set P(X) of a non-empty set X is a Boolean Algebra. with $\phi = 0$ and $X = 1$ and with the operations $\wedge$, $\vee$ and defined by $A \wedge B = A \cap B$; $A \vee B = A \cup B$ and $A' = X - A$, $\forall\, A, B \in P(X)$. 10

4. (a) Prove that a subset W of a vector space V(F) is a subspace if W is closed under vector addition and scalar multiplication. 10

(b) Show that the vectors (1, 1, – 1), (2, – 3, 5) and (– 2, 1, 4) of R^3 are linearly independent. 10

5. (a) Reduce the matris :

$$\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$$ to the normal form and find the rank. 10

(b) Solve the system of linear equations : 10

$$2x - y + 3z = 9$$
$$x + 3y - 2z = 1$$
$$5x - 3y + z = 2$$

6. (a) Find the characteristic equation of the matrix : 10

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

(b) Verify the Cayley Hamilton theorem for the matrix : 10

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -1 & 1 & 4 \end{bmatrix}$$

7. (a) Find the equation of a plane which cuts off the intercepts a, b, c from the coordinate axes. 10

(b) Find the equation of the plane passing through the point A(5, – 3, 2) and perpendicular to OA, where O is the origin. 10

8. (a) Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ are coplanar. 10

(b) Find the equation of the sphere whose center is (1, 2, – 1) and which passes through the point (1, – 2, 3). 10

 UG — BCA (F – 2001)

2019

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*Answer any **four** questions in which Q. No. 1 is compulsory.*

1. Answer **all** the questions of the following :

   $1 \times 10 = 20$

   (i) Form the composition table of the multiplicative group of cube roots of unity.

   (ii) Define the inverse of an element in a group.

   (iii) Define cosets and state Lagrange's theorem.

   (iv) Prove that in a ring R, $a(-b) = -(ab)$; $\forall a, b \in R$.

   (v) Define linear independence of vectors.

(vi) Define a basis of a vector space.

(vii) State the condition for a system of linear equations to have no solution.

(viii) Write down a matrix of order four in Eclon form.

(ix) Determine the relation between Cartesian and polar spherical coordinates of a point.

(x) Find the direction cosines of a line passing through the points (4,5,0) and (2,6,2).

2. (a) Prove that in a group G, $(ab)^{-1} = b^{-1}a^{-1}$; $\forall\, a, b \in G$. 10

(b) Prove that the identity element in a group is unique. 10

3. (a) Prove that a finite integral domain is a field. 10

(b) Prove that the power set P(X) of a non – empty set X is a Boolean Algebra with $\phi = 0$ and $X = 1$ and with the operations $\wedge$, $\vee$ and defined by, $A \wedge B = A \cap B$; $A \vee B = A \cup B$ and $A' = X - A$, $\forall\, A, B \in P(X)$. 10

4. (a) Prove that a subset W of a vector space V(F) is a subspace if W is closed under vector addition and scalar multiplication. 10

(b) Show that the vectors (1,1,–1), (2,–3,5) and (–2,1,4) of R^3 are linearly independent. 10

5. (a) Find the rank of the matrix 10

$$\begin{bmatrix} 1 & 5 & 7 & 1 \\ 3 & 12 & 1 & 1 \\ -1 & 2 & -13 & 1 \end{bmatrix}$$

(b) Solve the system of linear equations : 10

$$2x-y+3z=9$$
$$x+3y-2z=1$$
$$5x-3y+z=2$$

6. (a) Find the eigen values of the matrix

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix}.$$ 10

(b) Write down the matrix of the quadratic form $x_1^2-18x_1x_2+x_2^2$ and verify that it can be written as matrix products X^TAX. 10

7. (a) Find the angle between two lines whose direction cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2). 10

(b) Find the equation of the plane passing through the point A(5,–3,2) and perpendicular to OA, where O is the origin. 10

8. Find the shortest distance between two skew lines. 20